PHYS 320 ANALYTICAL MECHANICS

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Fall 2108

TODAY

Exam Thursday

Newton's Laws

Statics: next steps … trusses

Static Equilibrium

- Conditions of equilibrium:

$$\vec{F}_{net} = \sum_i \vec{F}_i = 0$$

$$\vec{\Gamma}_{net} = \sum_i \vec{\Gamma}_i = 0$$

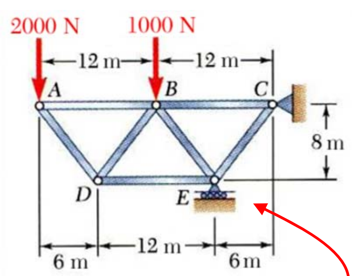
2nd Law

$$\vec{\tau}_{net} = \sum_i \vec{\tau}_i = 0$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

3rd Law

Static Equilibrium: Structures Example



note roller: no friction!

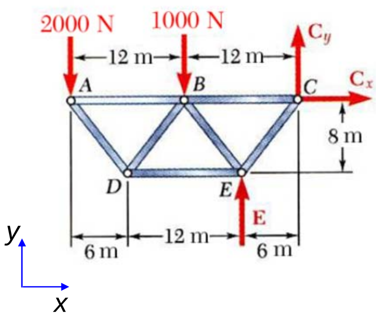
Find the force in each member of the truss!

Plan of attack:

- F-B diagram of entire truss; find reactions at supports
- Joint *A* has only two unknown member forces; find these first
- Next analyze joints D, B, & E.
- Use equilibrium conditions to check results at C.

Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example



$$\sum_{about\,C} \vec{\Gamma} = 0$$

$$\Rightarrow \vec{E} = 10{,}000N\ \hat{j}$$

$$\sum F_x = 0$$

$$\Rightarrow C_x = 0$$

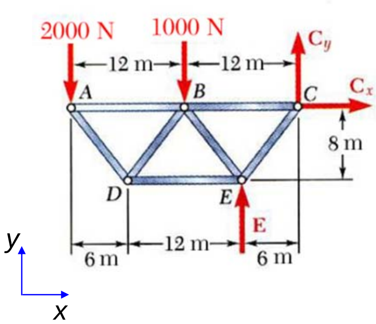
$$\sum F_y = 0$$

$$\Rightarrow C_y = -7{,}000N$$

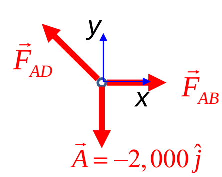
(we chose the wrong direction in diagram!)

Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example



Joint A:



$$\sum F_x = 0 = F_{AB} - F_{ADx}$$

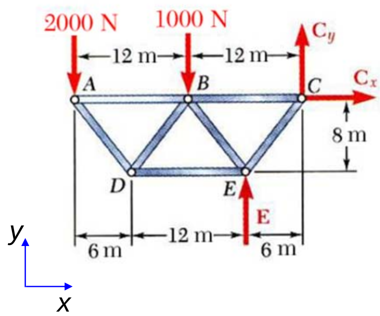
$$\sum F_y = 0 = F_{ADy} - A$$

$$\Rightarrow \begin{array}{l} F_{AD} = 2{,}500N \\ F_{AB} = 1{,}500N \end{array}$$

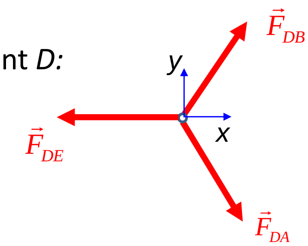
Member AD is in compression.
Member AB is in tension.

Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example



Joint *D:*



$$\sum F_x = 0 = F_{DBx} + F_{DAx} - F_{DE}$$

$$\sum F_y = 0 = F_{DBy} - F_{DAy}$$

$$F_{DB} = F_{DA} = 2,500N$$
$$F_{DE} = 3,000N$$

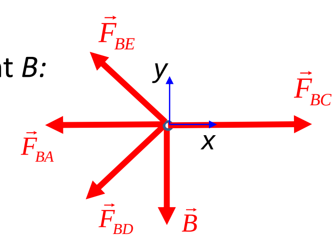
Member DB is in tension.
Member DE is in compression.

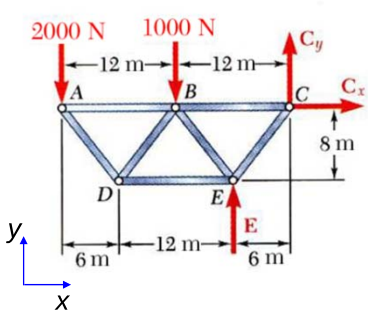
Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example

Here, we cannot be so certain about the direction of the two unknown forces.

Joint *B:*





$$\sum F_x = 0 = F_{BC} - F_{BA} - F_{BEx} - F_{BDx}$$

$$\sum F_y = 0 = F_{BEy} - F_{BDy}$$

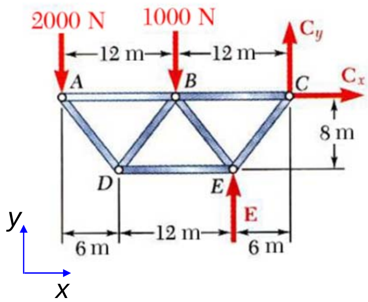
$$F_{BC} = 3,750N$$
$$F_{BE} = 5,250N$$

Member BC is in tension.
Member BE is in compression.

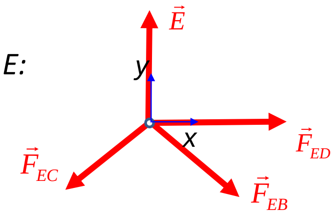
Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example

Here, we cannot be so certain about the direction of the two unknown forces.



Joint *E:*



$$\sum F_x = 0 = F_{ED} + F_{EBx} - F_{ECx}$$

$$\sum F_y = 0 = E - F_{EBy} - F_{ECy}$$

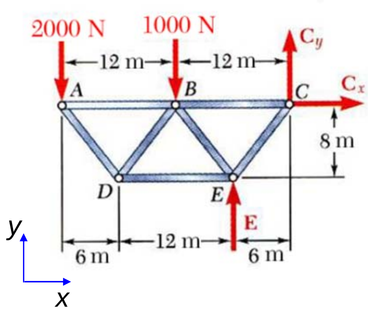
$$F_{EC} = 8,750N$$

Member EC is in compression.

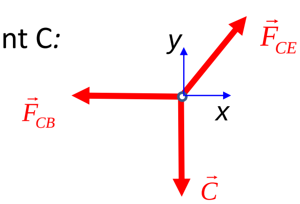
Ref: Beer & Johnston, Mechanics for Engineers: Statics

Static Equilibrium: Structures Example

Final check: we know all the member forces – see if they work for joint C!



Joint C:



$$\sum F_x = 0 = F_{CEx} - F_{CB}$$

$$\sum F_y = 0 = F_{CEy} - C$$

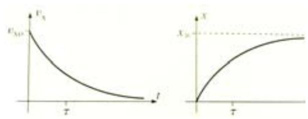
It Works!!

Ref: Beer & Johnston, Mechanics for Engineers: Statics

Linear Air Resistance:

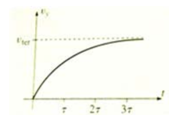
$$\vec{f}_{lin} = -b\vec{v}$$

Allows horizontal solutions



$$v_x(t) = v_{xo}e^{-bt/m} = v_{xo}e^{-t/\tau}$$

$$x(t) = v_{xo}\tau(1-e^{-bt/m}) = v_{xo}\tau(1-e^{-t/\tau})$$



Allows vertical solutions

$$v_y(t) = \frac{mg}{b} + (v_{yo} - \frac{mg}{b})e^{-bt/m} = v_{ter} + (v_{yo} - v_{ter})e^{-t/\tau}$$

$$y(t) = v_{ter}t + (v_{yo} - v_{ter})\tau(1-e^{-t/\tau})$$

$$v_{ter} \equiv \frac{mg}{b} \qquad \tau \equiv \frac{m}{b}$$

Linear Air Resistance: projectiles

$$x(t) = v_{xo}\tau(1-e^{-t/\tau})$$

$$y(t) = v_{ter}t + (v_{yo} - v_{ter})\tau(1-e^{-t/\tau})$$

$$v_{ter} \equiv \frac{mg}{b} \qquad \tau \equiv \frac{m}{b}$$

$$\Rightarrow \quad y = \frac{v_{yo} + v_{ter}}{v_{xo}}x + v_{ter}\tau\ln\left(1 - \frac{x}{v_{xo}\tau}\right)$$

The range can be approximated (Taylor series) as

$$R = x(y=0) \approx \frac{2v_{xo}v_{yo}}{g}\left(1 - \frac{4v_{yo}}{3v_{ter}}\right)$$

(low air resistance)